**Modeling Intelligent Reconfigurable Surfaces:**

We consider that we have a fixed coordinates system (x, y, z) defined by the location of the metasurface. This surface will be lying in the x-y plane of our coordinates system. The first element of this metasurface (upper left element) will be located at point (0, 0, 0) the origin of our coordinate system. The width of the surface will be spanning along the x direction, and the height will go along the y direction.

The z-axis will be the axis perpendicular to the surface, so the normal vector to the surface will be the unit vector along the z-axis.

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Figure : Metasurface model

We start by giving the following inputs to the model:

* Transmitter coordinates:
* Receiver coordinates:
* Index of refraction of free space
* For the transmitted signal:
* Transmitted Signal Frequency
* Transmitted Signal Amplitude
* Transmitted Signal Phase
* For the varactor:
* Element Resistance value
* Element bottom layer inductance
* Element top layer inductance
* Element effective capacitance
* Capacitance range that the varactor can produce.
* For the metasurface:
* Surface dimensions

Where is the number of elements along the width of the surface (in the x direction)

is the number of elements along the height of the surface (in the y direction)

* Elements size

Where the elements are considered as square with edge length .

* Element spacing

Where element spacing is the spacing between the edge of the first element and the edge of the second element.

Element spacing is the same in both x and y directions.

Then using the inputs, we previously mentioned, we can calculate some parameters as follows:

* Wavelength

Where is the speed of light

* The distance between the middle of 2 consecutive elements of the surface in both x and y directions:

The aim of this work is to create a Reconfigurable intelligent surface that can be reprogramed instantaneously in order to reflect an input signal from a given transmitter toward a known receiver.

The main working principle of this surface will follow the generalized Snell’s law for anomalous reflection in 3D space. This law is modeled by the following equations:

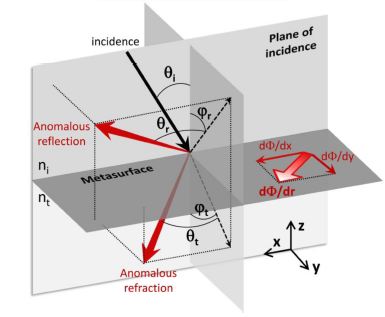


Figure : figure showing the angles used in the generalized Snell's law equations.

We denote by the projection of the reflected vector onto the plane perpendicular to the incident plane of incidence.

is the angle between the vector projection of the reflected vector and the z-axis.

is the angle between the reflected vector and its projection vector

We know the location of the transmitter and the receiver, and we want to properly reflect the signal from toward the receiver.

Based on this information, we can geometrically calculate the values of the reflection angles and to be able to calculate later the phase shifts needed for every element.

We consider that the transmitter in an omnidirectional antenna radiating in all directions. In our model we will discretize the propagation sphere by modeling it using equidistant rays generated from the transmitter and hitting each element of our surface. So, our model will only consider the part of the transmission sphere that will reach our surface, and it will be modeled as a ray hitting each element of our surface. So, the number of considered rays is equal to the number of elements on the surface. These rays will be represented by a matrix each entry of this matrix contain the coordinates of the incident ray hitting the corresponding element. These vectors will be calculated geometrically for every element using the following formula:

Since our goal is to reach the receiver with all the reflected rays, we will calculate the theoretical reflected rays considering all of them will reach the receiver. These rays will be represented by a matrix each entry of this matrix contain the coordinates of the reflected ray hitting the receiver. These vectors will be calculated geometrically for every element using the following formula:

After calculating the incident and the reflected vectors, the next step is to calculate the incident and the reflection angles. These angles are , and shown in the Figure 1 above.

For the incident angles, they will also be represented by a matrix, where each entry will represent the incident angle of the corresponding vector .

We know that is the angle between the incident vector and the normal to the plane which is in our case unit vector along the z-axis.

Then to calculate we will use the dot product equation:

We added a minus sign (-) to the incident vector to reverse the direction of the vector to have both and in the same direction so we can calculate the small angle between them which is .

Similarly, for and , we use the dot product notation to calculate these angles between the reflected vector and , and and respectively. We remind you that is the projection of the reflected vector onto the plane perpendicular to the incident plane of incidence.

and

To calculate the projection of the reflected vector onto the plane perpendicular to the incident plane of incidence, we will perform the following steps:

1. Find the normal vector of the plane of incidence.

We know that the incident vector is inside the plane of incidence, we also know that the unit vector along the z-axis is parallel to the plane of incidence. Then to find the normal vector to the plane of incidence, we will perform the cross product of the mentioned vectors.

1. Find the normal vector of the plane perpendicular to the plane of incidence (we will call this plane ).

We know that the unit vector along the z-axis is parallel to the plane . Additionally, since is perpendicular to the plane of incidence, then the vector normal to the plane of incidence will be also parallel to the plane . Moreover, from the previous step we know that vectors and are perpendicular to each other. Then the normal to the plane is the vector perpendicular to both and . To calculate this vector, we will perform the cross product between vectors and .

1. Calculate the projection of the reflected vector onto the plane .

To do so, we fist must calculate the projection of the reflected vector onto the vector which is the normal vector to the plane:

Then we calculate , the projection of the reflected vector onto the plane :

This process will be performed times computing the coordinates of vector the projection of the reflected vector onto the plane perpendicular to the incident plane for every incident vector. So, in the end we will have a matrix containing to the corresponding incident vector .

After successfully calculating the angles , and geometrically, now we can calculate the gradient phase shifts needed to reflect the transmitted signal toward the receiver location. To compute the phase shift gradient in both x and y direction we will use the generalized Snell’s law that we presented previously.

After this step we will have 2 matrices first the phase gradient along x direction, the second is the phase gradient along the y direction.

The next step is to use the and the matrices to find the phase shift for every element of the surface. To do so, we have to start thinking in the forward direction on how the gradients are calculated before thinking how to recover the phase shift function from its gradients. Then we will analyze the results of the forward thinking and apply it to the process of recovering .

The following equations are used to calculate the gradients of a function in both x and y direction:

will be a 2D array of size as follows:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
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To be able to calculate from its gradients and , we have to fix a starting point in the and used it along with the and values to calculate the rest of the values.

So, we fix the first value of and we assume it to be 0.

Now to calculate the rest of the values of the phase shifts matrix we can use an altered version of the derivatives formulas as follows:

Now we will write some of the equation to calculate the elements of the phase shifts matrix:

|  |  |
| --- | --- |
| *Row 1* | *Column 1* |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

If we notice from the equations above that row 1 can be calculating uniquely starting with and using only the gradient along x; .

Similarly, column 1 can be calculating uniquely starting with and using only the gradient along y; .

Then by calculating the unique values of row 1 from and column 1 from , we can use these values to calculates the rest of the phase shift values successively.

To calculate the other values, we will take as examples and

|  |  |  |
| --- | --- | --- |
|  |  |  |
| In x direction |  |  |
| In y direction |  |  |

As we can see we can calculate the same value of the phase shift function in two different ways using and but since we only have a single phase shift function so both ways should give the same result. In other words, in order for the phase shift gradient and matrices that we have to be the gradient of the phase shift function , then it does not matter which gradient it is used to calculate a given value since with both we will get the same results.

After this analysis, our strategy to find the phase shifts function matrix of from its gradients and is to calculate first two phase shifts matrices and using and respectively. Noting that in both matrices we have the same first column and first rows, where the first column is calculated starting with and using only the gradient along y; and the first row is calculated starting with and using only the gradient along x; .

Theoretically after these calculations, and should be exacly similar to each other’s, but given that we also have some imperfections when calculating initially the phase gradients and , we expect some error margin which will be shown by some differences in and . To solve this issue and to have finally one unique phase shift matrix, we will take the average of both matrices by summing them on element basis and dividing by 2. In the end, we will have the final phase shift function.

This function represents the phase shift that each element should apply on the incoming signal in order to have the desired reflection. will be a 2D matrix of size , where ech entry represents the phase shift required to be produced by the corresponding element of the metasurface.

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Figure : Required phase shifts.

Now that we have calculated the phase shift required by every element of the surface, we should procced in calculating the required capacitance for the element to produce this phase shift, and ultimately calculate the bias voltage that should be supplied to the varactor in order the produce the required capacitance.

We can find the phase shift of an element by checking its reflection coefficient .

Where:

* is the amplitude which practically will be in the range and considered as the resistive loss that the signal will endure when hitting a given element.
* is the actual phase shift calculated previously.
* is the impedance of free space.
* is the electrical impedance of the element. This impedance depends on the electronic model of a single element.

Below we will see the reflecting element electronic model and we will calculate .

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Figure : reflecting Element electronic model

We can see in Figure 2 the electronic model for a reflecting element. The impedance of this element can be calculated as follows:

Where:

* : effective resistance of element
* : bottom layer inductance of the element
* : top layer inductance of the element
* : effective capacitance of an element
* : angular frequency

Then to calculate the required capacitance value that will create the desired phase shift, we have to guess a value that will give the element a certain impedance and plugging this in the reflection coefficient equation we should have the angle equal to the desired phase shift. The range of values is decided by the model of the varactor used in the element and the capacitance range that it able to produce when given different voltages. This varactor should be sized to correspond to the predicted frequencies that will be used on this surface. In other words, the capacitance range that this varactor should produce should be in the exact range to be able to cover all the angles between .

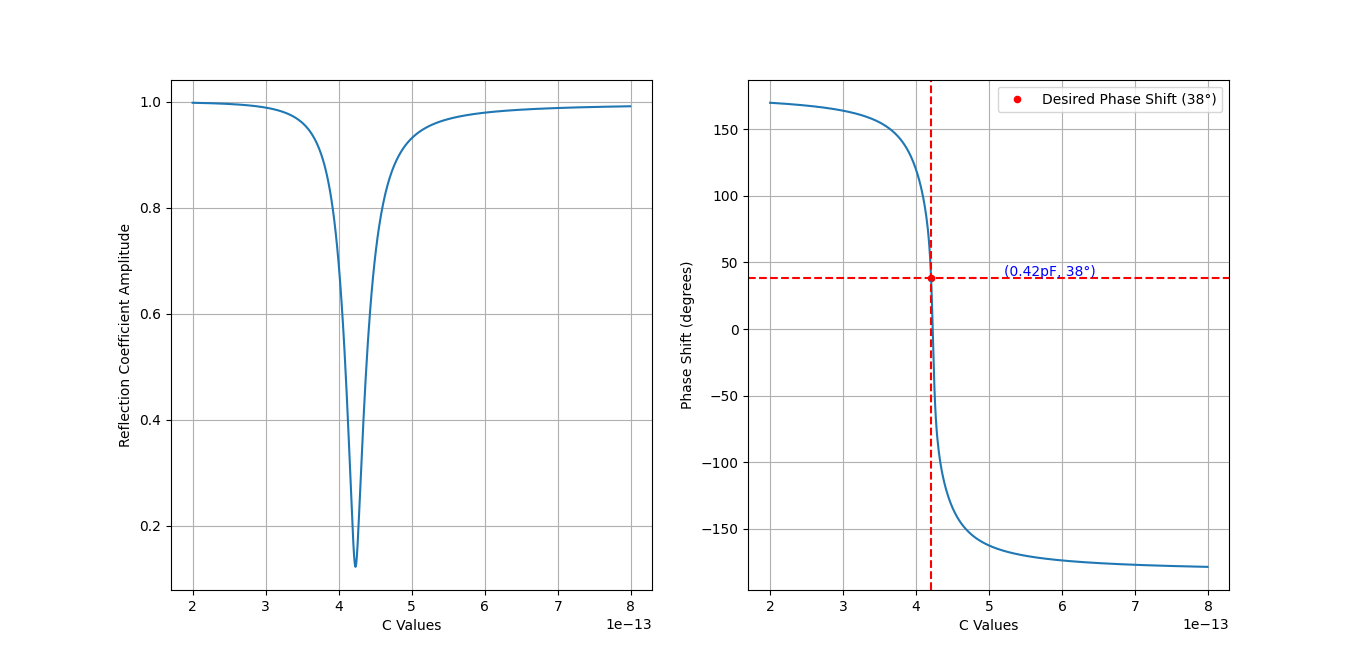


Figure : Amplitudes and Angles of the reflection coefficient with different C values (for a frequency f=10GHz)

In the Figure 3 above we can see the amplitudes and the angles of the reflection coefficient with different C values for a frequency . We can see that the capacitances are chosen in the right range for the reflection coefficient angle to cover a range very close to the range.

In what follows I will describe the strategy used to estimate the capacitance needed to create the desired phase shift:

1. First, as we spoke earlier, we will identify the range of that capacitance available. We should create a 1D matrix containing these capacitance values.
2. Using the available capacitance matrix, we will calculate the element impedance that could be created for each value of . (Knowing that the values of are constants).

Then the result will be an array of impedances having the same length as the array of available capacitances, and the value of the impedance on a given location of the array will corresponds to the value of the capacitance form the available capacitances array in the same location.

1. Using the achievable element impedances calculated earlier, we will now calculate the reflection coefficient of the element that could be achieved by the given element impedances.

The result will also be an array of reflection coefficients having the same length as the array of available capacitances, and the value of the reflection coefficient on a given location of the array will corresponds to the value of the capacitance form the available capacitances array in the same location.

1. In this step we will calculate the angles of every reflection coefficient we have in the elements achievable reflection coefficients array calculated in the previous step.

The result will also be an array of reflection coefficients angles having the same length as the array of available capacitances, and the value of the reflection coefficient angle on a given location of the array will corresponds to the value of the capacitance form the available capacitances array in the same location.

1. Now we have a connection between the capacitance and the phase shifts angles (which is basically the reflection coefficients angles array). The last thing left to do is to estimate the value of the capacitance C for a given phase shift. This estimation will be done by interpolation.

(Note: the more capacitance values we have in the initial available capacitance matrix, the more accurate the estimated capacitance will be a t the end of this method)

So finally, we take the phase shifts matrix that we calculated previously and apply the capacitance estimation method that we discussed earlier to find in the end the required capacitance for every element to achieve the desired phase shift. The result after this method will be a 2D matrix of size , where each entry represents the required capacitance to be tuned in the corresponding element of the metasurface to achieve its desired phase shift.

In the next step, we will calculate the real phase shift that will be actually produced by the surface based on the estimated capacitance value we calculated in the previous step. To do so, we must follow the following process:

1. Using the estimated elements capacitance matrix calculated earlier, we will calculate the real element impedance using its equation that we provided earlier. (Knowing that the values of are constants).

Then the result will be a 2D matrix of size , where each entry represents the actual impedance of the corresponding element on the metasurface.

1. Using the elements impedances calculated earlier, we will now calculate the reflection coefficient of the elements given their impedances.

Then the result will be a 2D matrix of size , where each entry represents the actual reflection coefficient of the corresponding element on the metasurface.

1. Now to find the real phase shifts, all what is left to do is to calculate the angle of each reflection coefficient in the real reflection coefficient matrix.

Then the result will be a 2D matrix of size , where each entry represents the real phase shifts that will be introduced by the corresponding element on the metasurface.

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Figure : Real Phase shifts

Then from the calculated real phase shifts of each element we can calculate the real reflection angles and of the reflected signal. To do so, we will derive in both x and y directions to get the gradient of the real phase shift used in the generalized Snell’s law.

To derive real phase shifts we will be using the following derivation formulas, considering the fact that the phase shifts should always be in the range, so even when subtracting 2 values of the , we should make sure that it belongs to the range .

As an example:

This equation means that the resultant of will be moved to the range.

Taking all of the above into account we will have the following equations to perform the derivatives:

Then after successfully finding the gradients of the real phase shifts , and . We can now use them to calculate the real reflected angles and based on the generalized Snell’s law of reflection. Then:

The results after this method will be two 2D matrix of size each, the first is the angle of the reflected ray from each element. The second is the angle of the reflected ray from each element.

By finding the real reflection angles and , and knowing the location of the transmitter, the location of the receiver and the location of every element of the metasurface, we can now calculate the real reflected vector. Then after calculating the reflected vector, we will check if it hits the antenna of the receiver successfully by checking if the endpoint of the reflected vector is inside the receiver antenna defined by its shape, its dimensions and its center which is the coordinates of the receiver that we defined in the beginning.

The first step is to find the real the projection of the reflected vector onto the plane perpendicular to the incident plane of incidence. We denote this plane by with its normal vector . The coordinates of this vector will be denoted by

We will also denote the unit vector of the projection of the reflected vector on the plane by

The origin of the vector is the point with the coordinates , and its extremity is the point with coordinates .

Then we can also denote the vector

We know the coordinates of the origin which are the coordinates of the element of the metasurface. Now we will be looking to find the coordinates of the extremity point of the unit vector .

We also know that is the angle between and therefore we can find :

Now we can write an equation for the plane :

We know that the point is in the plane , then we can write:

This is equation 1 which will be used the find the variables which are basically .

We can write this equation as:

To find the second equation we will use the formula of the norm of the vector which is a unit vector, then we have:

Combining these two equations, we get:

Assigning the following:

Now we have:

Solving this quadratic equation, we will have:

We have here 2 solutions for x, to choose between these solutions we will perform the following:

Project the receiver point onto the plane .

Compare the x component of the origin point of the vector , to the x component of the projection of the receiver.

now we found . Replace it in the y equation

we found

then we found the vector which is the unit vector of the vector that we are looking for. To find this vector from its unit vector, first we have to write in parametric form:

We are looking for the point of the vector that is on the plane of the receiver, in other words, we are looking for the point on the line having unit vector where

Now we replace this t in the equations of and to find their values.

Finally, we can find the reflected vector

After finding the projection of the reflected vector , the next step is to find the reflected vector using its reflection .

In this section we consider

The reflected vector

We want to find the coordinates of the reflected vector

We also know that the z coordinate of the vectors and will be the same:

We also know that

This is equation 1 used to find the reflected vector

To find the second equation we will use the dot product between and

Combining equations (1) and (2):

Assigning the following:

Now we have:

Solving this quadratic equation, we will have:

We have here 2 solutions for x, to choose between these solutions we will perform the following:

Where is the component of the origin of the reflected vector which is known (the x coordinate of the element of the metasurface).

Then we choose the solution which is closer to the x coordinate of the receiver, .

now we found . Replace it in the y equation

we found of the reflected vector

then to find the extremity point of this vector, or the point that will hit the plane of the receiver, we will do the following:

where the point is the origin of the reflected vector, which is the coordinates of point center of the element on the metasurface.

The next and the final step is to check if the reflected ray hits the receiver successfully. To do so we can simply define the receiver antenna shape, dimensions, and center point, then we can check it the point is inside or outside the area occupied by the antenna.

In the end, the result here will be a 2D Boolean “successful reflections ” matrix of size , where each entry will have a Boolean “True” value indicating that the reflected ray by the corresponding element is received corrected by the receiver antenna, or a Boolean “False” value indicating that the reflected ray by corresponding element misses the receiver antenna.

After determining the reflection model, by calculating the reflection angles that will dictate the path of the ray. We can now know if a given reflected ray will reach the receiver or not. The next step is to create the power model by calculating the effective power received by the receiver antenna.

The power model to be used in our case is similar to the 2-ray model, where we calculate the power received by a receiver coming from 2 paths, one line of sight path and another non-line of sight path where the signal is reflected by a surface and the reflected ray reaches the receiver. The 2-ray power model is the following:

In the power model for the IRS, we only care about the reflected part of the signal and not the line-of-sight part, since in case there is a line of sight between the transmitter and the receiver, the power reflected signal part will not be a huge addition to the total power received. So, we consider only the non-line-of-sight case where the received signal will only come from the reflected part of the transmitted signal by the metasurface. In this case, we will have the line-of-sight distance . Adapting the power equation, we will have:

In the equation above, we are summing over the powers of all reflected rays reaching the receiver.

The only missing thing from this equation is the fact that we have to disregard the power coming from the rays that will be reflected in an incorrect way and will not reach the receiver antenna. To take this issue into account, we will add the successful reflections matrix that we calculated previously. The power model will become:

When the reflected ray by the nth element hits the receiver successfully, so we include the power of this ray when calculating the received power. But when the reflected ray by the nth element misses the receiver successfully, so we multiply the power of this ray by “0” to ignore it and not take it into account when calculating the total power at the receiver.

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Figure : Power received vs number of elements

Finally, by calculating the reflection model and the power model, we have completed the Intelligent Reconfigurable Surface model. Now we have the power of the signal received. In the next part, we will calculate the power received by the receiver antenna when we do not have an intelligent metasurface and compare it with the IRS case to check the additional gain that the metasurface introduces. In this case the reflection will be governed by the original Snell’s law and the transmitted and the reflected rays will be in the same plane, so we have .

Given the location of the transmitter , the location of the receiver , the normal to the plane of incidence, (considering our initial assumption for a fixed coordinates system where the transmitter and the receiver are points in space, and the surface of incidence in the xy plane, (z=0) so the normal is the unit vector parallel to the z-axis ) we can calculate the angles and so a transmitted ray reaches the receiver.

We define the location of incidence at point . We have to find .

Then we have:

|  |  |  |
| --- | --- | --- |
|  | and |  |

Then we should find the point so the function above is closest to 0.

After finding the point , to find the angles :

The final part is to calculate the power received by the receiver antenna in a system without a metasurface, and we do not have a line-of-sight between the transmitter and the receiver. In this case the power will be:

Where is the reflection coefficient of the surface, this reflection coefficient will depend on the material of the surface, and it will be different for signals with perpendicular or parallel polarizations.

The result of this function will be the power of the signal received by the receiver antenna in the system without Intelligent Reconfigurable Surface.