We consider that we have a fixed coordinates system (x, y, z) defined by the location of the metasurface. This surface will be lying in the x-y plane of our coordinates system. The first element of this metasurface (upper left element) will be located at point (0, 0, 0) the origin of our coordinate system. The width of the surface will be spanning along the x direction, and the height will go along the y direction.

The z-axis will be the axis perpendicular to the surface, so the normal vector to the surface will be the unit vector along the z-axis.

We start by giving the following inputs to the model:

* Transmitter coordinates:
* Receiver coordinates:
* Index of refraction of free space
* For the transmitted signal:
* Transmitted Signal Frequency
* Transmitted Signal Amplitude
* Transmitted Signal Phase
* For the varactor:
* Element Resistance value
* Element bottom layer inductance
* Element top layer inductance
* Element effective capacitance
* Capacitance range that the varactor can produce.
* For the metasurface:
* Surface dimensions

Where is the number of elements along the width of the surface (in the x direction)

is the number of elements along the height of the surface (in the y direction)

* Elements size

Where the elements are considered as square with edge length .

* Element spacing

Where element spacing is the spacing between the edge of the first element and the edge of the second element.

Element spacing is the same in both x and y directions.

Then using the inputs we previously mentioned, we can calculate some parameters as follows:

* Wavelength

Where is the speed of light

* The distance between the middle of 2 consecutive elements of the surface in both x and y directions:

The aim of this work is to create a Reconfigurable intelligent surface that can be reprogramed instantaneously in order to reflect an input signal from a given transmitter toward a known receiver.

The main working principle of this surface will follow the generalized Snell’s law for anomalous reflection in 3D space. This law is modeled by the following equations:

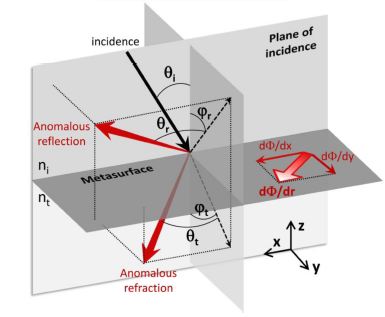


Figure 1: figure showing the angles used in the generalized Snell's law equations.

We denote by the projection of the reflected vector onto the plane perpendicular to the incident plane of incidence.

is the angle between the vector projection of the reflected vector and the z-axis.

is the angle between the reflected vector and its projection vector

We know the location of the transmitter and the receiver, and we want to properly reflect the signal from toward the receiver.

Based on this information, we can geometrically calculate the values of the reflection angles and to be able to calculate later the phase shifts needed for every element.

We consider that the transmitter in an omnidirectional antenna radiating in all directions. In our model we will discretize the propagation sphere by modeling it using equidistant rays generated from the transmitter and hitting each element of our surface. So, our model will only consider the part of the transmission sphere that will reach our surface, and it will be modeled as a rays hitting each element of our surface. So, the number of considered rays is equal to the number of elements on the surface. These rays will be represented by a matrix each entry of this matrix contain the coordinates of the incident ray hitting the corresponding element. These vectors will be calculated geometrically for every element using the following formula:

Since our goal is to reach the receiver with all the reflected rays, we will calculate the theoretical reflected rays considering all of them will reach the receiver. These rays will be represented by a matrix each entry of this matrix contain the coordinates of the reflected ray hitting the receiver. These vectors will be calculated geometrically for every element using the following formula:

After calculating the incident and the reflected vectors, the next step is to calculate the incident and the reflection angles. These angles are , and shown in the Figure 1 above.

For the incident angles, they will also be represented by a matrix, where each entry will represent the incident angle of the corresponding vector .

We know that is the angle between the incident vector and the normal to the plane which is in our case unit vector along the z-axis.

Then to calculate we will use the dot product equation:

We added a minus sign (-) to the incident vector to reverse the direction of the vector to have both and in the same direction so we can calculate the small angle between them which is .

Similarly, for and , we use the dot product notation to calculate these angles between the reflected vector and , and and respectively. We remind you that is the projection of the reflected vector onto the plane perpendicular to the incident plane of incidence.

and

To calculate the projection of the reflected vector onto the plane perpendicular to the incident plane of incidence, we will perform the following steps:

1. Find the normal vector of the plane of incidence.

We know that the incident vector is inside the plane of incidence, we also know that the unit vector along the z-axis is parallel to the plane of incidence. Then to find the normal vector to the plane of incidence, we will perform the cross product of the mentioned vectors.

1. Find the normal vector of the plane perpendicular to the plane of incidence (we will call this plane .

We know that the unit vector along the z-axis is parallel to the plane . Additionally, since is perpendicular to the plane of incidence, then the vector normal to the plane of incidence will be also parallel to the plane . Moreover, from the previous step we know that vectors and are perpendicular to each other. Then the normal to the plane is the vector perpendicular to both and . To calculate this vector, we will perform the cross product between vectors and .

1. Calculate the projection of the reflected vector onto the plane .

To do so, we fist must calculate the projection of the reflected vector onto the vector which is the normal vector to the plane:

Then we calculate , the projection of the reflected vector onto the plane :

This process will be performed times computing the coordinates of vector the projection of the reflected vector onto the plane perpendicular to the incident plane for every incident vector. So, in the end we will have a matrix containing to the corresponding incident vector .

After successfully calculating the angles , and geometrically, now we can calculate the gradient phase shifts needed to reflect the transmitted signal toward the receiver location. To compute the phase shift gradient in both x and y direction we will use the generalized Snell’s law that we presented previously.

After this step we will have 2 matrices first the phase gradient along x direction, the second is the phase gradient along the y direction.

The next step is to use the and the matrices to find the phase shift for every element of the surface. To do so, we have to start thinking in the forward direction on how the gradients are calculated before thinking how to recover the phase shift function from its gradients. Then we will analyze the results of the forward thinking and apply it to the process of recovering .

The following equations are used to calculate the gradients of a function in both x and y direction:

will be a 2D array of size as follows:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  | … |
|  |  |  |  |  |  |  |  | … |
|  |  |  |  |  |  |  |  | … |
|  |  |  |  |  |  |  |  | … |
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|  |  |  |  |  |  |  |  | … |
| . . . | . . . | . . . | . . . | . . . | . . . | . . . | . . . | … |

To be able to calculate from its gradients and , we have to fix a starting point in the and used it along with the and values to calculate the rest of the values.

So, we fix the first value of and we assume it to be 0.

Now to calculate the rest of the values of the phase shifts matrix we can an altered version of the derivatives formulas as follows:

Now we will write some of the equation to calculate the elements of the phase shifts matrix:

|  |  |
| --- | --- |
| *Row 1* | *Column 1* |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

If we notice from the equations above that row 1 can be calculating uniquely starting with and using only the gradient along x; .

Similarly, column 1 can be calculating uniquely starting with and using only the gradient along y; .

Then by calculating the unique values of row 1 from and column 1 from , we can use these values to calculates the rest of the phase shift values successively.

To calculate the other values, we will take as examples and

|  |  |  |
| --- | --- | --- |
|  |  |  |
| In x direction |  |  |
| In y direction |  |  |

As we can see we can calculate the same value of the phase shift function in two different ways using and but since we only have a single phase shift function so both ways should give the same result. In other words, in order for the phase shift gradient and matrices that we have to be the gradient of the phase shift function , then it does not matter which gradient it is used to calculate a given value since with both we will get the same results.

After this analysis, our strategy to find the phase shifts function matrix of from its gradients and is to calculate first two phase shifts matrices and using and respectively. Noting that in both matrices we have the same first column and first rows, where the first column is calculated starting with and using only the gradient along y; and the first row is calculated starting with and using only the gradient along x; .

Theoretically after these calculations, and should be exacly similar to each other’s, but given that we also have some imperfections when calculating initially the phase gradients and , we expect some error margin which will be shown by some differences in and . To solve this issue and to have finally one unique phase shift matrix, we will take the average of both matrices by summing them on element basis and dividing by 2. In the end, we will have the final phase shift function

This function represents the phase shift that each element should apply on the incoming signal in order to have the desired reflection. will be a 2D matrix of size , where ech entry represents the phase shift required to be produced by the corresponding element of the metasurface.

Now that we have calculated the phase shift required by every element of the surface, we should procced in calculating the required capacitance for the element to produce this phase shift, and ultimately calculate the bias voltage that should be supplied to the varactor in order the produce the required capacitance.

We can find the phase shift of an element by checking its reflection coefficient .

Where:

* is the amplitude which practically will be in the range and considered as the resistive loss that the signal will endure when hitting a given element.
* is the actual phase shift calculated previously.
* is the impedance of free space.
* is the electrical impedance of the element. This impedance depends on the electronic model of a single element.

Below we will see the reflecting element electronic model and we will calculate .

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Description automatically generated

Figure 2: reflecting Element electronic model

We can see in Figure 2 the electronic model for a reflecting element. The impedance of this element can be calculated as follows:

Where:

* : effective resistance of element
* : bottom layer inductance of the element
* : top layer inductance of the element
* : effective capacitance of an element
* : angular frequency

Then to calculate the required capacitance value that will create the desired phase shift, we have to guess a value that will give the element a certain impedance and plugging this in the reflection coefficient equation we should have the angle equal to the desired phase shift. The range of values is decided by the model of the varactor used in the element and the capacitance range that it able to produce when given different voltages. This varactor should be sized to correspond to the predicted frequencies that will be used on this surface. In other words, the capacitance range that this varactor should produce should in the exact range and should be able to cover all the angles between .

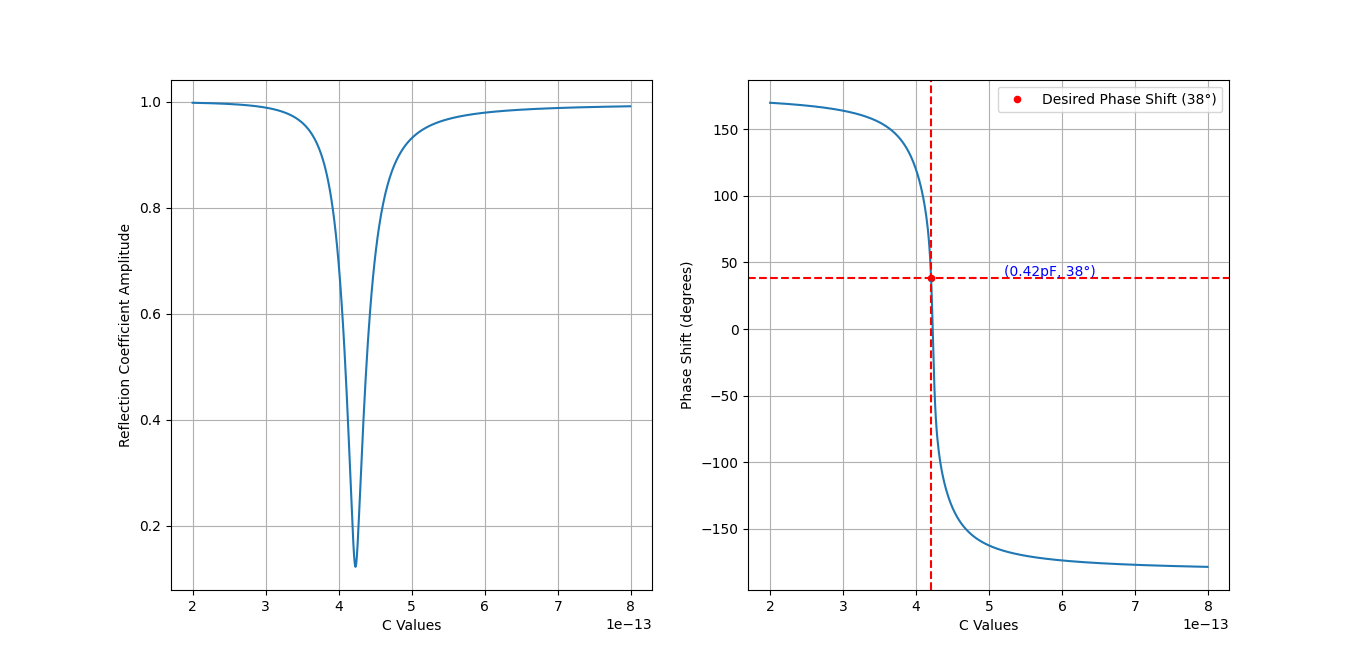


Figure 3: Amplitudes and Angles of the reflection coefficient with different C values (for a frequency f=10GHz)

In the Figure 3 above we can see the amplitudes and the angles of the reflection coefficient with different C values for a frequency . We can see that the capacitances are chosen in the right range for the reflection coefficient angle to cover the range.

In what follows I will describe the strategy used to estimate the capacitance needed to create the desired phase shift:

1. First, as we spoke earlier, we will identify the range of that capacitance available. We should create a 1D matrix containing the capacitance values.
2. Using the available capacitance matrix, we will calculate the element impedance that could be created for each value of . (Knowing that the values of are constants).

Then the result will be an array of impedances having the same length as the array of available capacitances, and the value of the impedance on a given location of the array will corresponds to the value of the capacitance form the available capacitances array in the same location.

1. Using the achievable element impedances calculated earlier, we will now calculate the reflection coefficient of the element that could be achieved by the given element impedances.

The result will also be an array of reflection coefficients having the same length as the array of available capacitances, and the value of the reflection coefficient on a given location of the array will corresponds to the value of the capacitance form the available capacitances array in the same location.

1. In this step we will calculate the angles of every reflection coefficient we have in the elements achievable reflection coefficients array calculated in the previous step.

The result will also be an array of reflection coefficients angles having the same length as the array of available capacitances, and the value of the reflection coefficient angle on a given location of the array will corresponds to the value of the capacitance form the available capacitances array in the same location.

1. Now we have a connection between the capacitance and the phase shifts angles (which is basically the reflection coefficients angles array). The last thing left to do is to estimate a value of the capacitance C for a given phase shift. This estimation will be done by interpolation.

(Note: the more capacitance values we have in the initial available capacitance matrix, the more accurate the estimated capacitance will be a t the end of this method)

So finally, we take the phase shifts matrix that we calculated previously and apply the capacitance estimation method that we discussed earlier to find in the end the required capacitance for every element to achieve the desired phase shift. The result after this method will be a 2D matrix of size , where each entry represents the required capacitance to be tuned in the corresponding element of the metasurface to achieve its desired phase shift.

In the next step we will calculate the real phase shift that will be actually produced by the surface based on the estimated capacitance value we calculated in the previous step. To do so, we must follow the following process:

1. Using the estimated elements capacitance matrix calculated earlier, we will calculate the real element impedance using its equation that we provided earlier. (Knowing that the values of are constants).

Then the result will be a 2D matrix of size , where each entry represents the actual impedance of the corresponding element on the metasurface.

1. Using the elements impedances calculated earlier, we will now calculate the reflection coefficient of the elements given their impedances.

Then the result will be a 2D matrix of size , where each entry represents the actual reflection coefficient of the corresponding element on the metasurface.

1. Now to find the real phase shifts, all what I left to do is to calculate the angle of each reflection coefficient in the real reflection coefficient matrix.

Then the result will be a 2D matrix of size , where each entry represents the real phase shifts that will be introduced by the corresponding element on the metasurface.

Then from the calculated real phase shifts of each element we can calculate the real reflection angles and of the reflected signal. To do so, we will derive in both x and y directions to get the gradient of the real phase shift used in the generalized Snell’s law.

To derive real phase shifts we will be using the following derivation formulas, considering the fact that the phase shifts should always be in the range, so even when subtracting 2 values of the , we should make sure that it belongs to the range .

As an example:

This equation means that the resultant of will be moved to the .

Taking all of the above into account we will have the following equations to perform the derivatives:

Then after successfully finding the gradients of the real phase shifts , and . We can now use then to calculate the real reflected angles and based on the generalized Snell’s law of reflection. Then: